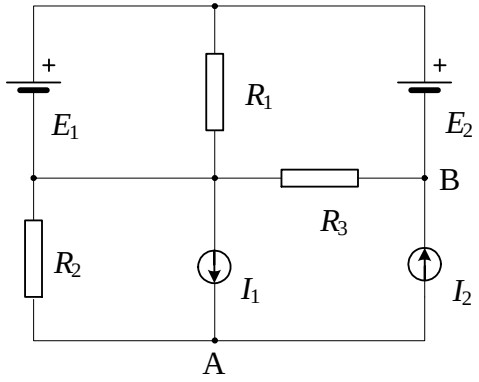
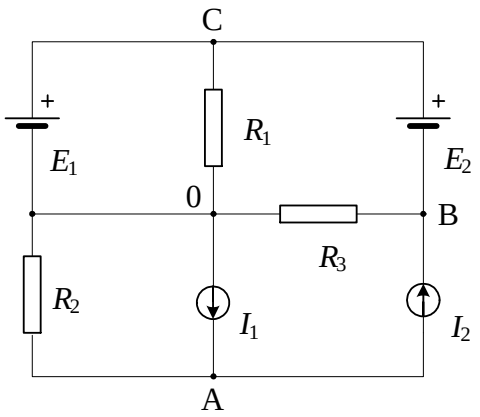
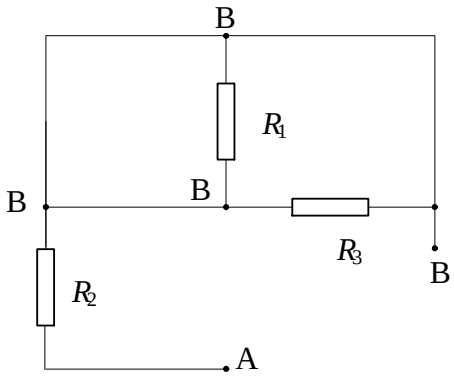
1. Vrednosti elemenata kola prikazanog na slici 1 su: R_1=1400 Ω, R_2= 100 Ω, R_3= 600 Ω, E_1 = 14 V, E_2 = 12 V, I_1 = 20 mA, I_2 =30 mA.

a) Kojom metodom se kolo sa slike može optimalno analizirati? Obrazložiti.

b) Odrediti ekvivalentni Tevenenov generator (optimalnom metodom) između tačaka A i B.







$n_g = 7$

$n_{č} = 4$

$n_{ks} = 7 - (4-1) = 4$

$n_{jks} = n_{ks} - n_{I_g} = 2$

$n_{nč} = 4 - 1 = 3$

$n_{jnč} = n_{nč} - n_E = 1$

$V_{\mathrm{B}} = -E_2 + E_1 = 2\,\mathrm{V}$

$V_{\mathrm{C}} = E_1 = 14\,\mathrm{V}$

$\left(\frac{1}{R_2}\right) \cdot V_{\mathrm{A}} = I_1 - I_2$

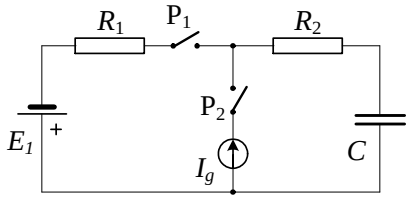
$V_{\mathrm{A}} = -1\,\mathrm{V}$

$E_{\mathrm{T}} = U_{\mathrm{AB}} = V_{\mathrm{A}} - V_{\mathrm{B}} = -3\,\mathrm{V}$

$R_{\mathrm{T}} = R_2$

$R_{\mathrm{T}} = 100\,\Omega$

2. Za kolo dato na slici 2 poznato je: Ig=10 mA, E_1=10 V R_1=R_2=500 Ω i C=1 μF. U trenutku t=0$^-$ oba prekidača su otvorena, a napon na kondenzatoru je jednak nuli. U trenutku t_0=0 prekidač P_2 se zatvori (P_1 ostaje otvoren), a u trenutku t_1=1ms zatvara se prekidač P_1, a otvara P_2.
a) Naći napon i struju kondenzatora u trenutku t_2=3ms.
b) Nacrtati oblik napona i struje na kondenzatoru.

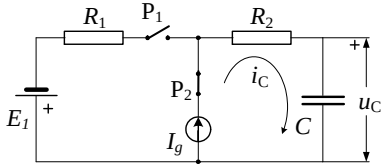


a) Stacionarno stanje:

$$i_C\left(t=0^-\right)=0$$

$$u_C\left(t=0^-\right)=0$$

P_2 zatvoren, P_1 otvoren:



$$i_C = I_g$$

$$i_C = C\cdot\frac{du_C}{dt} = I_g$$

$$\int du_C = \frac{I_g}{C}\cdot\int dt$$

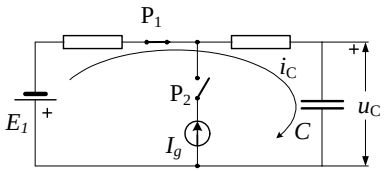
$$u_C = \frac{I_g}{C}\cdot t + K$$

$$u_C\left(t=0^-\right)=u_C\left(t=0^+\right)=0 \Rightarrow K=0$$

$$u_C(t)=\frac{I_g}{C}\cdot t$$

$$u_C\left(t_1^-\right)=U_1=10\,\mathrm{V}$$

P_1 zatvoren, P_2 otvoren:



$$\left(R_1+R_2\right)\cdot i_C + u_C = -E_1$$

$$i_C = C\cdot\frac{du_C}{dt}$$

$$\left(R_1+R_2\right)\cdot C\cdot\frac{du_C}{dt} + u_C = -E_1$$

$$u_C = u_{C_p} + u_{C_h}$$

$$u_{C_p} = -E_1$$

$$\int\frac{du_{C_h}}{u_{C_h}} = -\frac{1}{\left(R_1+R_2\right)\cdot C}\cdot\int dt$$

$$u_{C_h} = K\cdot e^{-\frac{1}{(R_1+R_2)C}\cdot t}$$

$$u_C = K\cdot e^{-\frac{1}{(R_1+R_2)C}\cdot t} - E_1$$

$$u_C\left(t=t_1^-\right)=u_C\left(t=t_1^+\right)=U_1 \Rightarrow$$

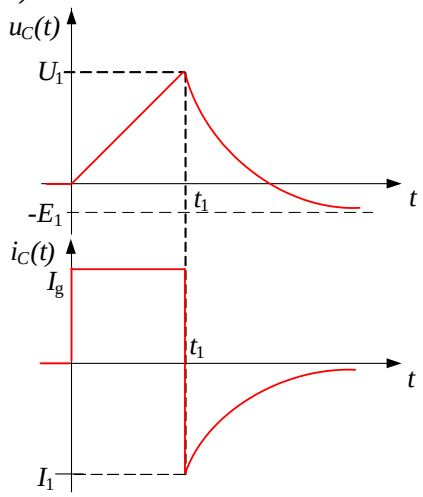
$$U_1 = K\cdot e^{-\frac{1}{(R_1+R_2)C}\cdot t_1} - E_1$$

$$K=\left(U_1+E_1\right)\cdot e^{\frac{1}{(R_1+R_2)C}\cdot t_1}$$

$$u_C = \left(U_1+E_1\right)\cdot e^{-\frac{(t-t_1)}{(R_1+R_2)C}} - E_1$$

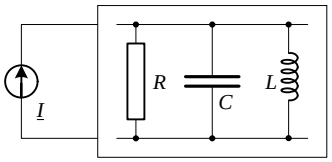
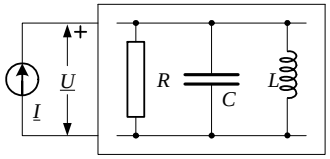
$$u_C\left(t=t_2\right)=-7{,}29\,\mathrm{V}$$

b)



3. Uređaj koji možemo predstaviti paralelno vezanim RLC kolom napaja se preko izvora naizmenične struje $\underline{I}$, kao što je prikazano na slici 3. Poznate su sledeće vrednosti elemenata: $\underline{I} = e^{j5\pi/6}$ mA na frekvenciji f = 2 kHz, R = 125 Ω, X_C = 125 Ω, X_L = 500 Ω.

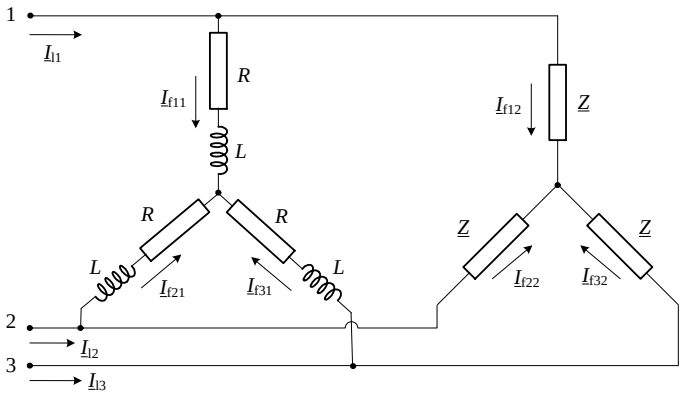
a) Izračunati struju uređaja u vremenskom domenu i napon na uređaju u kompleksnom i vremenskom domenu.
b) Kolika je kompleksna snaga koju uređaj troši?
c) Naći vrednost elementa koji treba vezati redno generatoru tako da faktor snage kola bude jednak jedinici.

<table>
<tr>
<td> 
</td>
<td> 
</td>
<td></td>
</tr>
<tr>
<td>a)
$\underline{Y} = \frac{1}{R} + j \cdot \left(\frac{1}{B_C} - \frac{1}{B_L} \right)$
$\underline{Y} = (8 + j \cdot 6)\text{mS}$
$\underline{Y} = 10 \cdot e^{j \cdot 36{,}87^\circ}\ \text{mS}$
$\underline{U} = \frac{\underline{I}}{\underline{Y}} = 0{,}1 \cdot e^{j \cdot 113{,}13^\circ}\ \text{V}$
$i(t) = \sqrt{2} \cdot \cos(12560 \cdot t + 150^\circ)\text{mA}$
$u(t) = 0{,}1 \cdot \sqrt{2} \cdot \cos(12560 \cdot t + 113{,}13^\circ)\text{V}$</td>
<td>b)
$\underline{S} = \underline{U} \cdot \underline{I}^*$
$\underline{S} = 0{,}1 \cdot e^{-j36{,}87^\circ}\ \text{mVA}$</td>
<td>c)
$Q = -60\,\text{mVA}$
$Q_k = -Q = 60\,\text{mVA} \Rightarrow$ zavojnica
$Q_k = I^2 \cdot \omega \cdot L$
$L = \frac{Q_k}{I^2 \cdot \omega} = 4{,}77\,\text{H}$</td>
</tr>
</table>

4. Tri identične impedanse vezane u zvezdu vrednosti R=10 Ω i L=100 mH i tri identične impedanse $\underline{Z}$=(20+j8)Ω takođe vezane u zvezdu priključene su na isti trofazni sistem linijskih napona U=380V i frekvencije f=50Hz (slika 4). Izračunati:

a) Sve linijske i fazne struje u kolu.

b) Ukupnu kompleksnu snagu koju ove impedanse troše.



a)

$$U_f = \frac{U}{\sqrt{3}} = 219{,}4\,\text{V}$$

$$\underline{Z}_1 = R + j\cdot\omega\cdot L = (10 + j\cdot 31{,}4)\Omega = 32{,}95\cdot e^{j\cdot 72{,}33^\circ}\Omega$$

$$\underline{U}_{f1} = U_f\cdot e^{-j0^\circ}$$

$$\underline{U}_{f2} = U_f\cdot e^{-j120^\circ}$$

$$\underline{U}_{f3} = U_f\cdot e^{-j240^\circ}$$

$$\underline{I}_{f11} = \frac{\underline{U}_{f1}}{\underline{Z}_1} = 6{,}66\cdot e^{-j\cdot 72{,}33^\circ}\text{A} = (2{,}02 - j\cdot 6{,}35)\text{A}$$

$$\underline{I}_{f21} = \frac{\underline{U}_{f2}}{\underline{Z}_1} = 6{,}66\cdot e^{-j\cdot 192{,}33^\circ}\text{A}$$

$$\underline{I}_{f31} = \frac{\underline{U}_{f3}}{\underline{Z}_1} = 6{,}66\cdot e^{-j\cdot 312{,}33^\circ}\text{A}$$

$$\underline{Z} = 21{,}54\cdot e^{j\cdot 21{,}8^\circ}\,\Omega$$

$$\underline{I}_{f12} = \frac{\underline{U}_{f1}}{\underline{Z}} = 10{,}18\cdot e^{-j\cdot 21{,}8^\circ}\text{A} = (9{,}45 - j\cdot 3{,}78)\text{A}$$

$$\underline{I}_{f22} = \frac{\underline{U}_{f2}}{\underline{Z}} = 10{,}18\cdot e^{-j\cdot 141{,}8^\circ}\text{A}$$

$$\underline{I}_{f32} = \frac{\underline{U}_{f3}}{\underline{Z}} = 10{,}18\cdot e^{-j\cdot 261{,}8^\circ}\text{A}$$

$$\underline{I}_{l1} = \underline{I}_{f11} + \underline{I}_{f12} = (11{,}47 - j\cdot 10{,}13)\text{A} = 15{,}3\cdot e^{-j\cdot 41{,}45^\circ}\text{A}$$

$$\underline{I}_{l2} = \underline{I}_{f21} + \underline{I}_{f22} = 15{,}3\cdot e^{-j\cdot 161{,}45^\circ}\text{A}$$

$$\underline{I}_{l3} = \underline{I}_{f31} + \underline{I}_{f32} = 15{,}3\cdot e^{-j\cdot 281{,}45^\circ}\text{A}$$

b)

$$\underline{S}_1 = 3\cdot U_f\cdot I_{f1}\cdot\cos\varphi_1 + j\cdot 3\cdot U_f\cdot I_{f1}\cdot\sin\varphi_1 = (1331 - j\cdot 4177)\text{VA}$$

$$\underline{S}_2 = 3\cdot U_f\cdot I_{f2}\cdot\cos\varphi_2 + j\cdot 3\cdot U_f\cdot I_{f2}\cdot\sin\varphi_2 = (6221 - j\cdot 2488)\text{VA}$$

$$\underline{S} = \underline{S}_1 + \underline{S}_2 = (7552 - j\cdot 6665)\text{VA}$$